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RISSE, Jr., Vilas Vernon, 1940-
DESIGN AND CONSTRUCTION OF MICROSTRIP
CIRCULATORS.

Iowa State University, Ph.D., 1969
Engineering, electrical

University Microfilms, Inc., Ann Arbor, Michigan

DESIGN AND CONSTRUCTION OF
MICROSTRIP CIRCULATORS

by

Vilas Vernon Risser, Jr.

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Major Subject: Electrical Engineering

Approved:

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1969

TABLE OF CONTENTS

	Page
LIST OF SYMBOLS	iii
INTRODUCTION	1
LITERATURE REVIEW	3
COMPARISON OF STRIP-LINE AND MICROSTRIP CIRCULATORS	20
DESIGN AND CONSTRUCTION PROGRAM	33
Use of the Desired Circulator	34
Microstrip Parameters	36
Ferrite Parameters	47
Determination of the Proper Bias Field	55
Matching the Ports	60
Measurement of Circulator Characteristics	63
CONCLUSIONS	68
BIBLIOGRAPHY	70
ACKNOWLEDGMENTS	75
APPENDIX	76

LIST OF SYMBOLS

α_c	conductor loss in microstrip
α_d	dielectric loss in microstrip
β^+	lossless propagation constant in $+\phi$ direction
β^-	lossless propagation constant in $-\phi$ direction
B	input susceptance of ferrite disk
C	capacity of microstrip line
ΔH	linewidth of ferrite in oersteds
∇	vector laplacian
δ'	fractional frequency splitting required for an admittance phase angle of 30° at the resonator
2δ	bandwidth of device expressed in per cent
\underline{E}	vector electric field
E_i or H_i	field component in i^{th} coordinate direction
ϵ_0	dielectric constant of free space (scalar)
ϵ	relative dielectric constant (scalar)
ϵ_{eff}	effective dielectric constant of microstrip
G_R	input conductance of ferrite disk
\underline{H}	vector magnetic field
H_{dc}	value in oersteds of external magnetic bias field
H_i	internal magnetic field in the ferrite
H_{res}	bias field value required for ferromagnetic resonance
h	vertical distance between microstrip conductors
In. L.	insertion loss of device measured in decibels
Isol.	value of port isolation measured in decibels
J_n	n^{th} order Bessel function of the first kind

k	wave propagation number
κ	off-diagonal component of $\underline{\mu}$
$4\pi M_0$	value of magnetization measured in the laboratory
$4\pi M_s$	saturation magnetization of ferrite
n	number of azimuthal variations around ferrite disk
N	demagnetizing factor
Φ	potential associated with microstrip lines
2ψ	angle subtended by width of microstrip line
q	effective filling fraction used to evaluate ϵ_{eff} and μ'_{eff} for microstrip
Q	charge per unit length of microstrip line
r	radius of conducting disk over ferrite
R	ferrite disk radius
R_s	surface resistivity in ohms per square meter
$\rho-\phi-z$	regular cylindrical coordinate symbols
$\tan \delta$	loss tangent of dielectric
θ	angle associated with input admittance
$\underline{\mu}$	tensor permeability
μ_0	permeability of free space (scalar)
μ	on-diagonal component of $\underline{\mu}$
μ_{eff}	effective permeability of ferrite
μ'_{eff}	effective permeability modified for microstrip
VSWR	voltage standing wave ratio
ω	r.f. driving frequency measured in radians ($\omega = 2\pi f$)
ω_0	center frequency of circulator operation measured in radians
W	width of microstrip line

$x-y-z$	regular cartesian coordinate symbols
Y_{in}	input admittance of ferrite disk
Y_R	admittance of ferrite disk at band edge frequencies
Z	free space characteristic impedance
Z_0	characteristic impedance of microstrip

LIST OF FIGURES

	Page
Figure 1. Strip-line circulator model	6
Figure 2. Microstrip transmission line	14
Figure 3. Microstrip circulator model	21
Figure 4. Standing wave pattern in ferrite disk for either strip-line or microstrip circulator, $H_{dc} = 0$	27
Figure 5. Standing wave pattern rotated 30° to isolate one port	30
Figure 6. Equivalent circuit of counter-rotating mode admittance	31
Figure 7. In-line transition from coax to microstrip	40
Figure 8. Microstrip conductor geometry near end of lines	41
Figure 9. Input VSWR versus frequency	43
Figure 10. Different conductor geometries used on ferrite disk	44
Figure 11. Resonance curve showing different regions of operation for a fixed rf frequency	52
Figure 12. Insertion loss versus H_{dc}	58
Figure 13. Port admittance plot	59
Figure 14. Transformer-ferrite connection	62
Figure 15. Necessary laboratory equipment for complete testing program	64
Figure 16. Picture of complete microstrip circulator	66
Figure 17. Characteristics of C-band microstrip circulator	67
Figure 18. Variation of operating frequency with conductor disk radius	79

INTRODUCTION

A device which is finding extensive use in the microwave industry is the ferrite circulator. Basically a circulator is a ferrite loaded symmetrical junction of three or more transmission lines. It has the property of transferring power from the incident port to the next adjacent port and isolating all other ports. The nonreciprocal characteristics of the ferrite, under the influence of proper magnetic bias fields, make this action possible. Theoretically, a circulator can have any number of ports. However, the difficulties of realizing an n -port circulator increase as n increases. Practically 3 and 4-port circulators have been produced. The 3-port or Y-junction circulator is the most common and is of major interest here.

The possibility of such a device was proposed in 1954 (14). A scatter matrix analysis of a lossless reciprocal 3-port junction was made and it was shown that not all ports of the junction could be matched. However, if the requirement for reciprocity were lifted, it would be possible theoretically to match all three ports. Further, it was shown that if a lossless nonreciprocal 3-port junction could be matched it would behave as a perfect circulator (14, 34). Since the nonreciprocal properties of ferrites were being studied at the time this material was used to load a 3-port junction. After some experimentation circulator action was achieved.

As with most devices, the first models were crude, bulky and inefficient. Improvements were made and, as the trend of the microwave industry was toward miniaturization, the circulator was soon being adapted from waveguide to strip-line. With the ability to process ferrite materials to smaller dimensions, and to use more efficient means of providing d.c. magnetic bias it was possible to reduce the size still further, and adapt the element to the unsymmetrical microstrip transmission line.

Building microstrip circulators is a new process using state of the art techniques. There is a great deal of research activity in this area today and much work needs to be done. It is the purpose of this paper to describe one such research effort. A broad study program was undertaken, including ferrite parameters, microstrip characteristics, and the circulation mechanism. From this base a systematic design plan was formulated and carried to successful completion. This design plan, used to obtain several working models of microstrip circulators, will be discussed. Emphasis will be placed on the problems likely to be encountered in the laboratory, but enough theoretical background will be given to justify each assumption or clarify each step.

LITERATURE REVIEW

The microstrip circulator is the culmination of two different disciplines -- propagation of energy by microstrip transmission lines, and ferrite circulator design and operation. An attempt will be made to trace the development of both microstrip and the circulator. Not all papers pertaining to the subject will be covered. Likewise, some of the papers will be covered more thoroughly than others. The chief objective of this review is to furnish sufficient background to enable the reader to grasp the problem.

In 1955, Carlin (14) was the first to show that any lossless nonreciprocal 3-port junction which can be matched acts as a perfect circulator. This basic premise caused attention to be focused on producing a practical circulator. This effort culminated in the development of the circulator although no phenomenological theory of operation was available.

Much research effort was expended on the relatively new device and several papers concerning circulators were written. In 1956, Treuhaft (43) wrote a theoretical paper, in which he pointed out the similarities between the scattering matrix for circulators and the cyclic substitution of the group theory.

Auld (7) using the mathematics developed in Treuhaft's paper, considered the problem of synthesizing symmetrical waveguide circulators. Milano, Saunders and Davis (33) analyzed a strip-line Y-circulator using the scatter matrix

approach. This paper also gives some performance data for a circulator using two Yig disks polarized above resonance¹. At a given frequency and for a given ferrite material the disk diameter and the applied bias field were varied until all ports were as nearly matched as possible. This assured circulator action.

Another active worker in the ferrite circulator area was Yoshida (49). This author gives some helpful hints on circulator design in his brief reports. Chait and Curry (16) published a report containing performance data for a geometrically symmetric 3-port waveguide junction containing a triangular ferrite wedge. An attempt to describe the circulation phenomena was also made by proposing that the fields in the magnetized ferrite were displaced in such a way that the energy could be guided from one port to the adjacent one.

This field displacement theory and others were used to explain circulator phenomena, but it was not until 1961 that the boundary value problem for a strip-line Y-circulator was solved.

Bosma (11) published in June of that year the first of his two classic papers in which he gave extensive consideration to the circulator problem. He noted that as long as three fold symmetry was maintained the junction could be

¹In this report above resonance operation implies that the applied magnetic bias field is greater than the bias required for ferromagnetic resonance.

loaded with any shape ferrite. However, for mathematical convenience he analyzed the simplest geometry of two circular ferrite disks on either side of the center conductor of the strip-line junction. Figure 1. Starting with Maxwell's equations for plane wave propagation in an infinite sheet of gyrotropic medium, Bosma solves for the electromagnetic fields. Several assumptions are made to reduce the boundary value problem to a tractable form. Although some of these assumptions seem to be restrictive the results obtained did corroborate them. The main assumptions made were 1) the electric field is in the axial direction only, 2) there is no variation of electric field in the axial direction, 3) the edges of the ferrite disks can be considered as magnetic walls, 4) the only boundary condition necessary restricts the azimuthal magnetic field to be zero at the ferrite disk edge and 5) since there is symmetry in balanced strip-line one ferrite disk acts as a mirror image of the other and only one disk need be considered.

Bosma obtains from his analysis two equations necessary for circulation. These equations depend both upon the ferrite parameters and the fields in the junction region. Solutions of these two complex equations give conditions which must be met to assure circulation.

In this comprehensive paper Bosma also proposed a qualitative explanation of why circulation occurred. His explanation is based on the electric field distribution in the

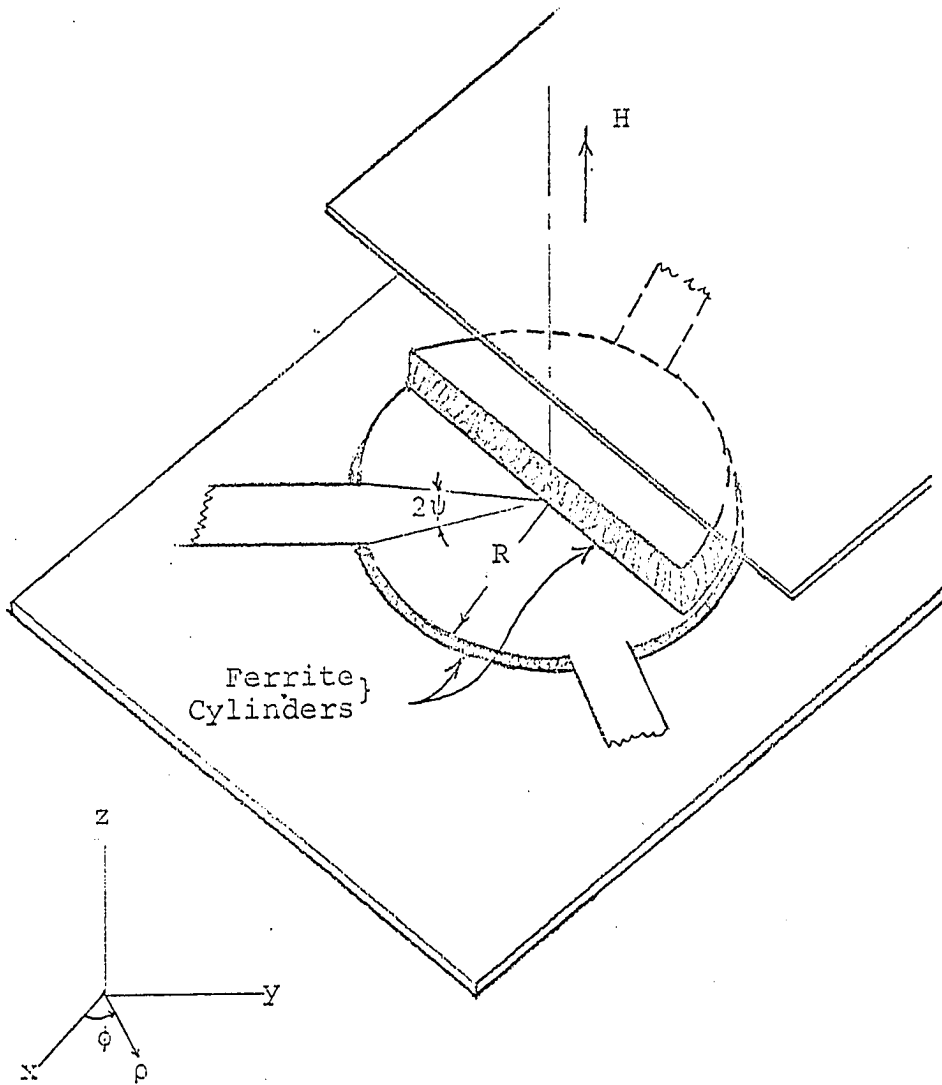


Figure 1. Strip-line circulator model

ferrite disks. Plots of constant amplitude and phase of the electric field were made and it was shown that most of the energy was concentrated in the region between the input and output ports. Bosma explained the circulation mechanism by considering that a plane wave incident upon a ferrite disk will be split into two elliptically polarized waves, one traveling in the $+\phi$ direction and one in the $-\phi$ direction. The $+$ and $-$ signs correspond to the waves traveling in the same direction, and to waves oppositely directed to the natural electron spin in the ferrite. If there is no magnetic field across the ferrite the $+\phi$ and $-\phi$ traveling waves will be degenerate, i.e. they will travel around the disk with the same propagation velocity. When a d.c. magnetic field is applied the waves no longer travel around the disk with the same velocity and the two counter-rotating waves will provide destructive interference at some point on the circumference of the disk. The d.c. magnetic field can be varied until this null is produced at the port to be isolated.

This paper went far toward explaining the phenomena of circulator action. It was also the first solution to the boundary value problem presented for the strip-line Y-circulator. The theoretical design equations obtained did not violate any known experimental results obtained by the author or by earlier experimenters. The paper also provided a firm foundation for further work on circulators.

Following Bosma's first paper research on circulators

was accelerated. Many novel design methods were reported. One notable attempt to miniaturize the circulator was reported by Clark and Brown (18). These authors reported on a coaxial Y-junction circulator using a gadolinium substituted yttrium iron garnet post as the gyrotropic material. This particular type garnet was found to provide stable operation over a wide range of frequencies and temperatures. Clark and Brown also discussed various methods of changing the operating characteristics of ferrite circulators. The two parameters most often varied are the external magnetic field and the ferrite disk diameter. These authors also mentioned the possibility of changing the height of the ferrite cylinder, using "an adjustable ground plane below the ferrite, and shielding the ferrite post by an adjustable height dielectric sleeve. Data was presented for a coaxial Y-circulator operating over temperature ranges from -60°F to 300°F .

Skomal (39) emphasized the physical behavior of the ferrite circulator. It was proposed that the ferrite cylinder could support propagation of a bound surface wave. The idea of the incident wave splitting into two oppositely directed components was applied -- assuming that the energy was contained in a tightly bound surface wave. As the axial magnetic d.c. field was applied the propagation speed of the oppositely directed waves differed. The attenuation of each component was assumed to be the same. Skomal then proposed that the d.c. magnetic field, the saturation magnetization, and the

ferrite cylinder diameter could be chosen so as to give constructive and destructive interference of the oppositely directed components of the surface wave. These parameters could then be adjusted to provide circulator action. One condition that must be met, according to this source, is that the circumferential distance between ports must equal λ_f the wave length of the surface wave. Skomal draws several interesting conclusions from his work with circulators. One is that for any fixed diameter ferrite operating with a fixed d.c. magnetic field there are two frequencies which will show circulation -- one above and one below resonance. Furthermore, the circulation will be in opposite directions at the two frequencies. Another point is that below resonance operation allows a ferrite with a broader line width to be used without degrading the insertion loss of the circulator. Also for below resonance operation, the optimum applied d.c. magnetic field decreases as $4\pi M_s$, the saturation magnetization, increases. He also verified that wider circulator bandwidths occur for below resonance operation.

Shortly before Bosma's second paper Davies and Cohen (20) published a paper in which they gave some substance to the validity of the circulation equations presented in Bosma's first paper. They used a scatter matrix formulation and derived the scatter matrix elements for the 3-port circulator. Then they solved the corresponding eigenvalue problem and showed that if the eigenvalues were spaced around the unit

circle 120° apart two equations arise which are equivalent to the circulation equations of Bosma. However, they offer some advantages, both in interpretation and computation, because many difficulties caused by singularities in Bosma's equations are removed. The authors concluded that several apparent modes of operation were possible depending upon the pair of degenerate resonant modes set up in the gyromagnetic disks. The choice of mode of operation affects the physical parameters and performance of the circulator.

In 1963, Bosma (12) followed up his first work on circulators by presenting another paper on circulation at U H F. He drew on the conclusions of Davies and Cohen and realized that the circulators he described in his first paper were operating in an inefficient higher order mode. He also reformulated the basic boundary value problem and used a Green's function method of solution. In this comprehensive report Bosma also proposes a method for broadbanding circulators. Then he concludes his second paper with a revised discussion of the circulation mechanism in the ferrite loaded junction.

In his two papers Bosma laid much of the ground work for understanding the circulator mechanism. A thorough knowledge of his two definitive papers is a must for anyone planning to work with ferrite circulators.

Following this work a flurry of activity was apparent in circulator research. Many authors proposed different design methods for accomplishing various ferrite circulators (10, 30).

The techniques reported varied according to the purpose of the circulator. Although Bosma's circulators were above resonance the area below resonance was reported in many works.

One major interest of people designing circulators was obtaining wider bandwidths. A comprehensive study was reported by Simon (38). He bases his study on the concept that building a 3-port circulator is essentially a matching process. He took impedance measurements on circulators with different ferrite materials and different biasing levels. When the impedance points began to coalesce about the real axis on the Smith chart a matching structure from ferrite to strip-line could be designed. Some of the points discussed in this paper are the effect of the saturation magnetization on the circulator impedance, the effect of ferrite geometry on impedance characteristics, the fact that the dielectric loss of the ferrimagnetic materials is quite noticeable for circulators operating below resonance, and that strip-line width has little effect on impedance characteristics. Simon was able to achieve octave bandwidth circulators by effectively matching the 3-port junction.

Anderson (1) addresses the problem of broadbanding the circulator by using external matching circuits. He reaches an interesting conclusion regarding the limiting number of external matching structures which will improve performance over any specified bandwidth.

A helpful paper which extended Bosma's theory and gave

design techniques was written by Fay and Comstock (22). These men used an engineering approach to the problem of constructing ferrite circulators. Bringing the power of circuit theory to bear on the problem they deduced a design program which is widely used in the industry today. The particulars of this design plan will be discussed elsewhere in this report. Let it simply be stated here that the authors designed several strip-line circulators and achieved quite good correlation between theoretical and experimental results.

Other papers on circulators were published. Weiss (45) worked with a symmetrical 3-port ring circulator composed of T-junctions and nonreciprocal phase shifters. This author also discussed the question of how efficiently the volume of ferrite is used in present devices.

Various other papers gave different ideas and methods of optimizing the operation of ferrite circulators. Konishi (30) worked with a lumped element circulator and reported on a synthesis theory of such. Bonfeld, Linn and Omori (10) reported on a circulator utilizing only one magnetic pole piece. Good circulation characteristics were obtained with this compact unit over a small bandwidth.

Although progress has been made in the circulator theory the design of waveguide and strip-line circulators is still not perfected. Realizing a working circulator is an engineering problem. However, if the problems which will be encountered in the laboratory are to be solved a thorough knowledge

of circulator history is necessary. The listing of papers reviewed here is not comprehensive but an investigation of these papers will lead to other sources. Some general background material related to the problem can be found in many textbooks (17, 19, 26, 29, 32, 34, 40, 41, 42, 44).

This report concerns microstrip circulators. Ordinarily not much attention would be focused on the type of transmission system used, but since microstrip transmission of energy is a problem in its own right a brief review of microstrip history seems in order.

The trend in recent years has been to decrease the size of electrical components. With the advent of integrated circuits and thin film techniques a type of transmission line known as microstrip began to attract attention. This type of unbalanced transmission line, Figure 2, was first described by Assadourian and Rimai (6). In their paper they analyzed the properties of microstrip assuming TEM mode propagation. For the TEM mode to exist the top conductor, of finite width, must be imbedded in an infinite homogeneous dielectric above a ground plane. Practical microstrip does not meet these conditions. However, Assadourian and Rimai suggested that if the bottom conductor is more than three times the width of the top conductor most of the energy is concentrated in the dielectric between the conductors. Although their solution is not rigorous it gives good results and most of the theories of microstrip include the assumption of TEM mode propagation.

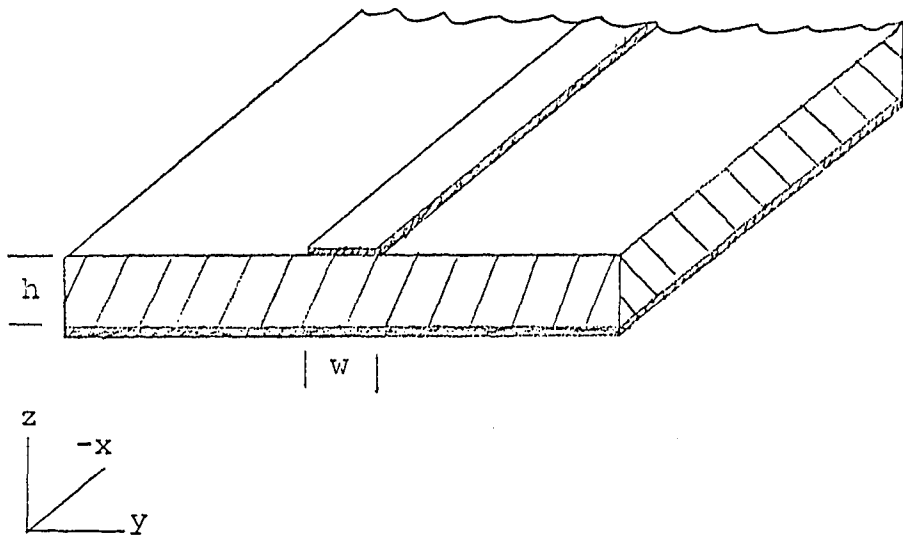


Figure 2. Microstrip transmission line

The important parameters of microstrip are the characteristic impedance Z_0 , the guide wavelength λ_g , and the losses due to the dielectric and conductors. The authors discussed each of these parameters and evaluated them for several different configurations.

Despite the approximations made this work provided the first attempt to analyze the microstrip line. The solution is only good for wide strips, but gives good results in this area. As will be seen shortly a rigorous solution of the microstrip boundary value problem soon becomes too unwieldy to provide useful information for the engineer interested in microstrip design.

In the same year as Assadourian and Rimai's work two other papers appeared concerning microstrip. Grieg and Engelman (23) gave results that generally agree with those of (6). Kostrize (31), on the other hand was concerned with the usefulness of microstrip for microwave components. This paper describes transitions from microstrip to coax as well as descriptions of various microwave components such as pads, loads, directional couplers, and ring structures.

Following this series of papers microstrip drew some laboratory investigations, but the industry was not yet ready for microstrip components. Although these papers laid the ground work it was a decade before the advent of practical microstrip components. However, some useful papers were published in that decade. Arditi (2, 3, 4) published three

papers concerning his work with microstrip. These papers describe the investigation undertaken to determine if microstrip could be used as a substitute for coax and waveguide in a complete system. The work was performed at 3-5 Ghz. The effects of transducers, bends and other obstacles in the line were studied.

All the work up to this time had been based on assumptions similar to those made by Assadourian and Rimai. Black and Higgins (9) solved the difficult problem of determining the characteristics of a microstrip line with three variable parameters, both conductor widths and the line spacing. They used a complex variable technique whereby several transformations were used and a rigorous solution was obtained by evaluating complicated Schwartz-Christoffel integrals. The fact that laborious and difficult evaluation of these integrals is required does not detract from their solution. However, the engineer interested in design of microstrip components still awaited a handy method of evaluating the parameters of microstrip.

Brodwin (13) studied transmission characteristics for a microstrip line in which the substrate material was gyrotopic. His analysis and experiments closely paralleled those for a ferrite filled waveguide in a longitudinal magnetic field.

With the publications by Wheeler (46, 47) microstrip attracted new interest. Wheeler emphasizes the synthesis, versus analysis, of microstrip from the design engineer's

point of view. Such a design engineer is interested in the relationship between the structural aspects of the line and the electrical characteristics. The usual problem is what shape ratio -- conductor width over line spacing -- should be used to obtain a microstrip line with a certain characteristic impedance. Wheeler uses a conformal mapping technique but certain simplifying approximations are made and the results are given in terms of the trigonometric and exponential functions instead of the less used and more complex elliptic functions. Wheeler's work has been the basis for several papers which give easy to use charts for obtaining the shape ratio of microstrip for a desired characteristic impedance (27, 35).

With the increased interest in integrated circuitry in the last few years the interest in microstrip continued to grow. Hylltin (28) reported results of various line widths laid down on the usual SiO_2 dielectric used for integrated circuitry. This high dielectric substrate more nearly binds the r.f. energy to the substrate.

Caulton, Hughes and Sobol (15) have a readable paper which provides some background theory, then uses Wheeler's paper as a basis to provide easy to use design charts. Also discussed is the non-trivial problem of transitions from microstrip to associated circuitry.

Much work is being done with microstrip today, both theoretical and experimental. Although the exact boundary

value problem of the unbalanced transmission line remains untenable, attempts to obtain better approximations to the line characteristics continue (48). Since the trend of the microwave industry is to minimize the size and weight of various components this work is expected to continue.

Within the past year several preliminary reports of research being conducted on microstrip circulators have been published. Hershenov (24, 25) has reported on several different configurations. Also Dunn and Domenico have worked on these devices (21). Most of these devices have been constructed by drilling a hole in a dielectric substrate and inserting a ferrite disk. Then conductors are deposited or etched over the disk. There has been no theoretical analysis published concerning the microstrip circulator but some people believe it will behave phenomenologically very much like the strip-line model. Most researchers believe the device holds great promise, but that mass production is still several years away.

This was not intended to be an exhaustive review of the literature concerning microstrip and circulator theory. It was intended to acquaint the reader with some of the work which has preceded this paper in both fields so that he might appreciate some of the problems involved. Since much work is being done in these areas today any literature review such as this is necessarily outdated before it is published. However, the reader who is starting to research in either the field of

microstrip or of circulators should find in these papers a proper basis on which to proceed with his study.

COMPARISON OF STRIP-LINE AND MICROSTRIP CIRCULATORS

The objective of the program was to design and construct a C-band microstrip circulator which operated below resonance. A ferrite disk of proper dimensions was to be imbedded in a suitable dielectric substrate. Figure 3. Quarter-wave transformers were to be used to match the ports. The design was to be based on the strip-line circulator design outlined by Fay and Comstock (22).

In order to determine what assumptions remain valid for microstrip a brief discussion of the strip-line analysis will be given here. Reference will be made to the two models shown in Figures 1 and 3.

First consider the magnetic field. It is assumed that the ferrite is placed in a uniform external field and that the magnetic dipoles are precessing in the uniform precessional mode. In strip-line where two magnets are used this is very nearly so. In microstrip where only one pole piece is feasible the uniformity of the field is questionable. Using a magnetic probe edge effects were observed in the magnetic field above the ferrite. Attempts to place a magnetic keeper above the ferrite produced interference with the rf fields. This is one reason why a low saturation material is desired for microstrip circulators. It was also found that the magnet diameter should be greater than the ferrite disk diameter to minimize the edge effects.

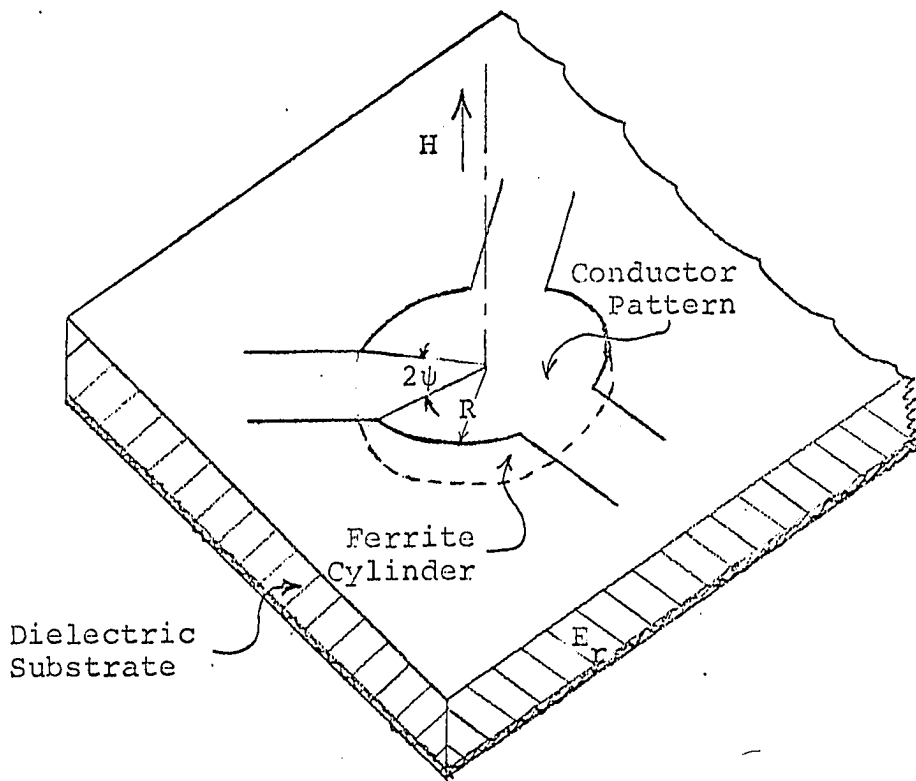


Figure 3. Microstrip circulator model

Another assumption which simplifies the strip-line solution is that there is no electric field variation in the z-direction. This, coupled with the assumption that there is a component of electric field in the axial direction only, allows the solution of the fields in the ferrite. The appropriate Maxwell's equations are:

$$(1a) \quad \underline{\nabla} \times \underline{E} = j\omega\mu_0 \underline{\mu H}$$

$$(1b) \quad \underline{\nabla} \times \underline{H} = -j\omega\epsilon_0 \epsilon \underline{E}$$

$$(1c) \quad \underline{\nabla} \cdot \underline{E} = 0$$

$$(1d) \quad \underline{\nabla} \cdot \underline{\mu H} = 0$$

where

$$(1e) \quad \underline{\mu} = \begin{Bmatrix} \mu & -j\kappa & 0 \\ j\kappa & \mu & 0 \\ 0 & 0 & 1 \end{Bmatrix}$$

Using cylindrical coordinates and the assumptions mentioned above manipulation of Maxwell's equations result in the Helmholtz equation for E_z

$$(2a) \quad \left(\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + k^2 \right) E_z = 0$$

where

$$(2b) \quad k^2 = \omega^2 \epsilon_0 \mu_0 \epsilon \left(\frac{\mu^2 - \kappa^2}{\mu} \right) = \omega^2 \epsilon_0 \mu_0 \epsilon \mu_{eff}$$

This equation has solutions, depending on the number of azimuthal variations n , given by

$$(3) \quad E_{zn} = J_n(k\rho) (a_{+n} e^{jn\phi} + a_{-n} e^{-jn\phi}).$$

The magnetic field components can be found in terms of E_z from the following equations:

$$(4a) \quad H_\rho = \left(\frac{1}{\rho} \frac{\partial E_z}{\partial \phi} - j \frac{\kappa}{\mu} \frac{\partial E_z}{\partial \rho} \right) / j\omega\mu_0 \left(\frac{\mu^2 - \kappa^2}{\mu} \right)$$

$$(4b) \quad H_\phi = - \left(\frac{\partial E_z}{\partial \rho} + j \frac{\kappa}{\rho\mu} \frac{\partial E_z}{\partial \phi} \right) / j\omega\mu_0 \left(\frac{\mu^2 - \kappa^2}{\mu} \right)$$

$$(4c) \quad H_z = 0$$

These results are

$$(4a') \quad H_\rho = - \sqrt{\frac{\epsilon\epsilon_0}{\mu_0 \left(\frac{\mu^2 - \kappa^2}{\mu} \right)}} \{ a_{+n} e^{jn\phi} \left(\frac{\kappa}{\mu} J_{n-1}(k\rho) - \frac{n}{k\rho} J_n(k\rho) \left(1 - \frac{\kappa}{\mu} \right) \right) \right. \\ \left. + a_{-n} e^{-jn\phi} \left(\frac{\kappa}{\mu} J_{n-1}(k\rho) + \frac{n}{k\rho} J_n(k\rho) \left(1 + \frac{\kappa}{\mu} \right) \right) \right\}$$

$$(4b') \quad H_\phi = j \sqrt{\frac{\epsilon\epsilon_0}{\mu_0 \left(\frac{\mu^2 - \kappa^2}{\mu} \right)}} \{ a_{+n} e^{jn\phi} \left(J_{n-1}(k\rho) - \frac{n J_n(k\rho)}{k\rho} \left(1 + \frac{\kappa}{\mu} \right) \right) \right. \\ \left. + a_{-n} e^{-jn\phi} \left(J_{n-1}(k\rho) - \frac{n J_n(k\rho)}{k\rho} \left(1 - \frac{\kappa}{\mu} \right) \right) \right\}$$

$$(4c') \quad H_z = 0$$

This solution for the fields is based on only one ferrite disk. It is agreed that since there is field symmetry and geometrical symmetry about the center conductor plane in the strip-line, the ferrite disks act as mirror images of one another. It seems reasonable then that the fields in the microstrip case would be described by similar expressions. Obvious changes, such as in the effective dielectric constant, will have to be made but the basic field configuration in the ferrite will remain the same.

Matching the boundary conditions in either the strip-line or microstrip model is a formidable task. In the strip-line problem Bosma (11, 12) has argued that forcing the azimuthal magnetic field H_ϕ to be zero at the ferrite disk edge is sufficient to describe the normal modes set up in the isolated disk case. ($\psi = 0$, Figure 1). When external transmission lines are connected to the central conductor this condition does not hold at the ports. The problem is still tractable however if H_ϕ is assumed zero everywhere on the edge of the ferrite except the ports.

With the external transmission lines attached let the boundary conditions for $\rho = R$ be given by:

$$(5a) \quad \text{Port \#1} \quad H_\phi = H_1, \quad E_z = E_1$$

$$(5b) \quad \text{Port \#2} \quad H_{\phi} = H_1 \quad E_z = -E_1$$

$$(5c) \quad \text{Port \#3} \quad H_{\phi} = 0 \quad E_z = 0$$

$$(5d) \quad \text{Elsewhere} \quad H_{\phi} = 0$$

Using these conditions, and considering only the $n = 1$ modes the amplitudes a_+ and a_- can be evaluated as:

$$(6a) \quad a_+ = \frac{E_1}{2J_1(kR)} \left(1 + \frac{j}{\sqrt{3}}\right)$$

$$(6b) \quad a_- = \frac{E_1}{2J_1(kR)} \left(1 - \frac{j}{\sqrt{3}}\right)$$

Then the fields of interest are:

$$(7) \quad E_{z1} = E_1 \frac{J_1(k\rho)}{J_1(kR)} \left(\cos\phi - \frac{\sin\phi}{\sqrt{3}}\right)$$

$$(8) \quad H_{\phi 1} = j \sqrt{\frac{-\epsilon\epsilon_0}{\mu(\frac{\mu^2 - \kappa^2}{\mu})}} \frac{E_1}{2J_1(kR)} \left\{ \left(1 + \frac{j}{\sqrt{3}}\right) \left(J_0(kR) - \frac{J_1(kR)}{kR}\right) \right. \\ \left. \left(1 + \frac{\kappa}{\mu}\right) e^{j\phi} + \left(1 - \frac{j}{\sqrt{3}}\right) \left(J_0(kR) - \frac{J_1(kR)}{kR}\right) \left(1 - \frac{\kappa}{\mu}\right) e^{-j\phi} \right\}$$

In order for the magnetic field to satisfy the boundary conditions

$$(9) \quad \left(J_0(kR) - \frac{J_1(kR)}{kR}\right) \left(1 + \frac{\kappa}{\mu}\right) = 0 = \left(J_0(kR) - \frac{J_1(kR)}{kR}\right) \left(1 - \frac{\kappa}{\mu}\right)$$

must be satisfied. This is so only if

$$(10) \quad (kR)_{1,1} = 1.84.$$

This equation proves useful in determining the required ferrite disk diameter.

Using the fields found as discussed above as a basis the phenomenological description of what is occurring in the ferrite loaded junction can be outlined. Similar phenomena is thought to take place in both strip-line and microstrip.

Let it be assumed that a linearly polarized sinusoidally time varying wave is incident on the ferrite cylinder. The electric field vector is perpendicular to the plane of the ferrite disk and the magnetic field vector is parallel to, and in the plane of, the ferrite.

It is assumed that the ferrite broad faces are completely covered with conductor leaving only the disk edges open to radiation. When the d.c. magnetic field is zero this plane wave will set up a standing wave pattern as shown in Figure 4.

It must be emphasized that this is but one mode which is set up in the ferrite disk. This is the dominant mode, but when circulator action is considered it is necessary to include all possible modes in which the electric field vector is perpendicular to the plane of the ferrite disk. Although all of these modes are not involved in the power handling process they are necessary when matching the fields in the ferrite to

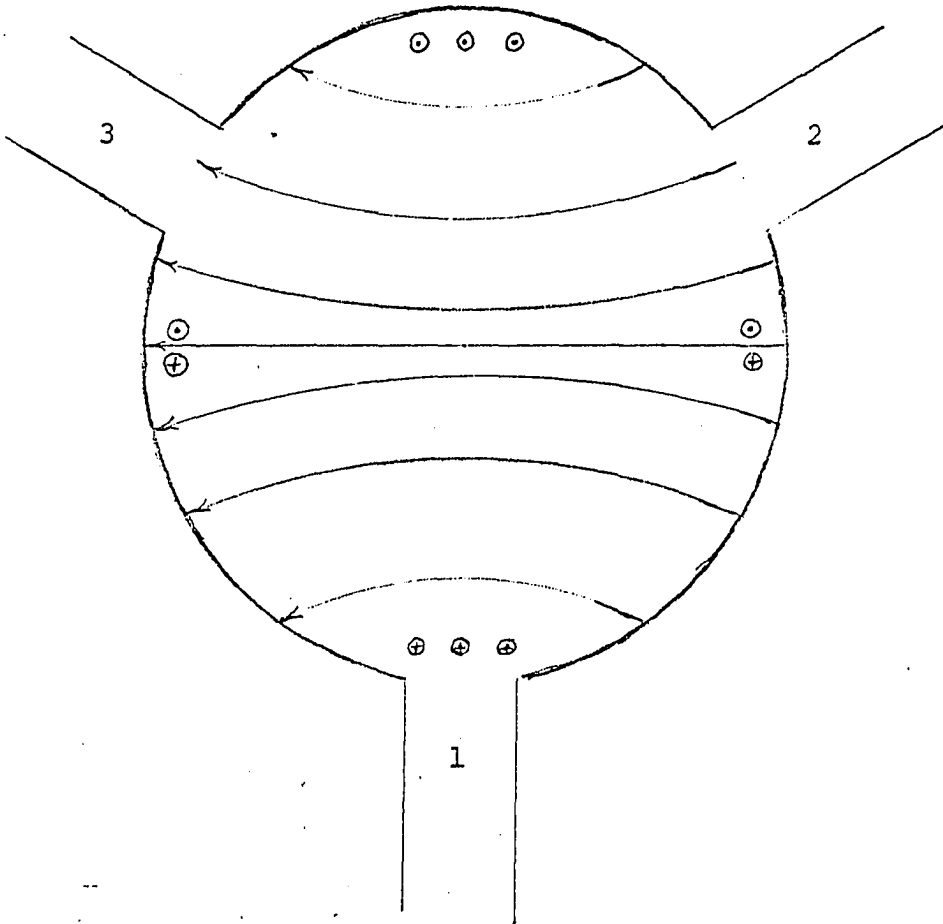


Figure 4. Standing wave pattern in ferrite disk for either strip-line or microstrip circulator, $H_{dc} = 0$

the fields present in the surrounding dielectric. This is true for the 3-port Y-junction circulator. It is thought that an attempt to excite some of these higher order modes, more than one variation in the ϕ direction, might be beneficial in the design of multiport circulators.

The pattern of Figure 4 can be thought of as a superposition of two elliptically polarized counter-rotating field patterns or modes with propagation constants β^+ and β^- . The propagation will be assumed lossless. In the absence of a magnetic bias field β^+ and β^- will have the same value. Ports II and III will see voltages which are 180° out of phase with the input voltage and one half the value of the voltage at port I.

When a magnetic bias field is placed across the ferrite disk the tensor characteristics of the ferrite are observed. The magnetic dipoles produced by the spinning electrons will tend to precess around the magnetic bias field vector. This affects the propagation of the counter-rotating waves such that β^+ and β^- are no longer equal. Each of these modes will have its own resonant frequency which will depend on the saturation magnetization and the radius of the ferrite disk. The plus mode, rotating in the same direction as the precessing dipoles, will have a higher resonant frequency than the minus rotating mode. The components of electric and magnetic field will arrive at ports II and III out of phase and will add to give a composite value. The standing wave pattern will, in

effect, be rotated by an amount which depends on the applied magnetic field. For the 3-port Y-junction circulator the field pattern should be rotated 30° in the plus direction to isolate port III. Figure 5. The system then acts like a transmission device between ports I and II. Note that circulation is in the direction of the minus rotating mode if the applied bias field is less than the field required for resonance of the ferrite. If $H_{dc} > H_{res}$ the direction of circulation is reversed. Since there is complete symmetry in the device any port can be chosen as the input port with similar results. In this discussion it was tacitly assumed that all ports were matched, which is a condition for circulation. Otherwise the problem would be complicated by multiple reflections.

In effect, the rotation of the standing wave pattern by 30° can be thought of in the following manner. Suppose that each counter-rotating mode has a mode admittance. The lumped element equivalent circuit of such an admittance might be given by Figure 6. Since the port admittance is desired to be real at the operating frequency it would be beneficial if the reactive components of the two counter-rotating mode impedances would cancel. This can be adjusted by choosing the proper bias field, ferrite radius, and saturation magnetization. All of these ferrite parameters must be considered individually, yet they are interrelated in their effect on the operating characteristics.

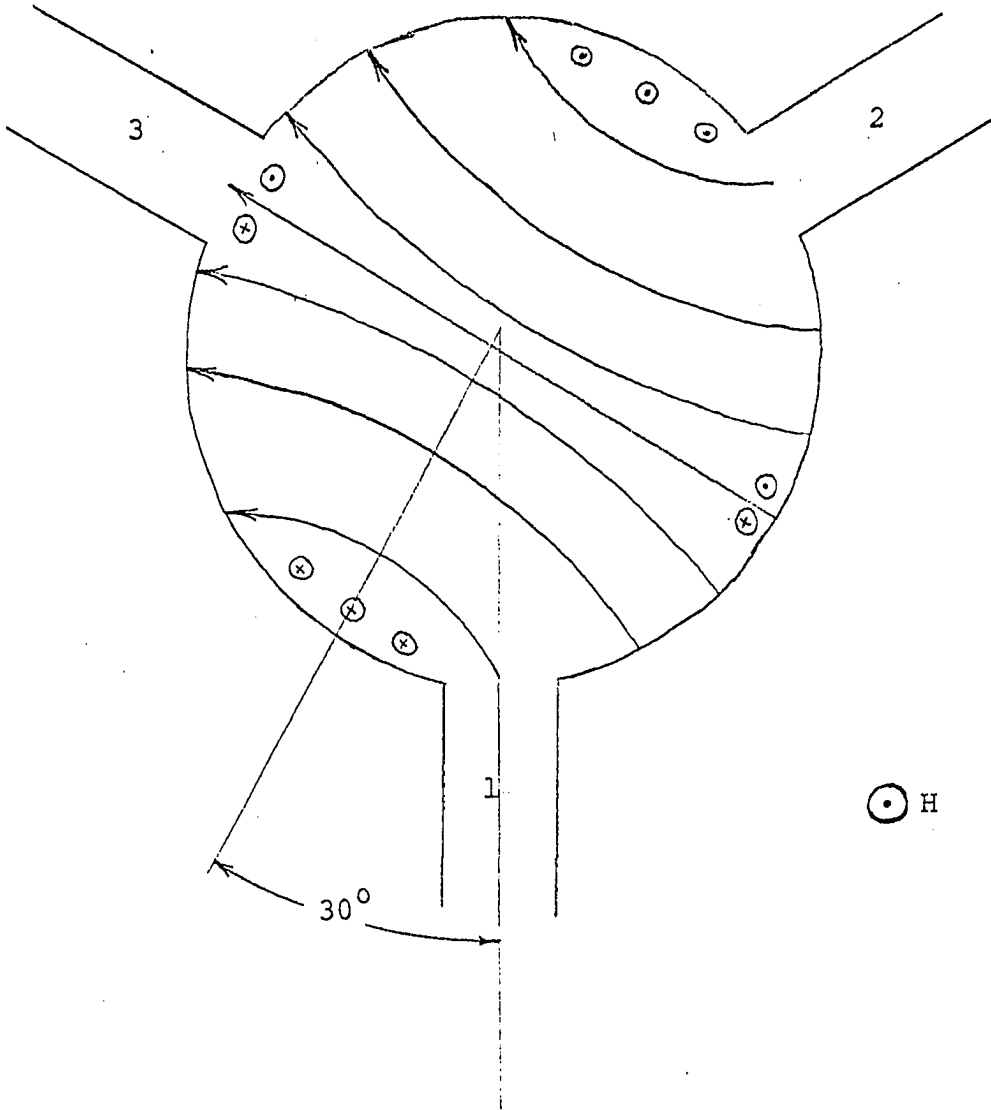


Figure 5. Standing wave pattern rotated 30° to isolate one port

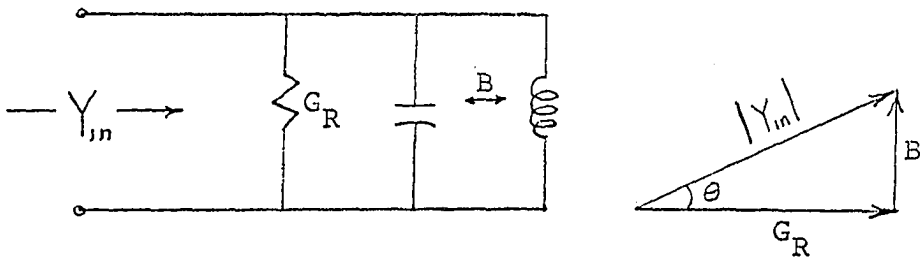


Figure 6. Equivalent circuit of counter-rotating mode admittance

If the circulator is to be operated below resonance a ferrite material with low saturation magnetization $4\pi M_s$ is desired. This will then require a smaller external magnet to produce the desired bias field. Although more will be said concerning ferrite selection later, let it be assumed here that a ferrite with a certain $4\pi M_s$ has been selected. The question of importance is finding the proper diameter of the ferrite disk. Utilizing Equation 10 the ferrite disk radius is

$$(11) \quad R = 1.84 / \omega \sqrt{\epsilon \epsilon_0 \mu_0 \mu_{\text{eff}}}.$$

Fay and Comstock (22) give curves which are useful for obtaining μ_{eff} . Knowing this quantity, the permittivity, and the driving frequency allows the calculation of k and thus the ferrite radius.

Fay and Comstock also give a method of obtaining the ferrite disk thickness. It is thought that for microstrip, where the substrate is usually thin, it is not necessary to calculate an exact value of disk thickness. Even though the obtainable bandwidth may decrease slightly as the thickness of the ferrite decreases, it is thought that this is not a sizeable effect. In choosing the ferrite disk a more practical consideration is dielectric substrate thickness.

With these comments in mind a design plan patterned on the outline just presented will be proposed. This plan was used successfully to obtain useful microstrip circulators.

DESIGN AND CONSTRUCTION PROGRAM

The purpose of this section is to outline the step by step procedure which was used in the design and development of several microstrip circulators. It is thought to be a straight-forward and systematic approach to an engineering problem of considerable interest. Whenever possible, theory has been reproduced to qualify or substantiate experimental procedure. Suggestions are sometimes made for improvement of the procedures used. These, for the most part, are based on experience gained in the laboratory. The sensitive areas are often the ones over which no design equations are available. Unfortunately, small variations in these areas can often produce sizeable changes in the operating characteristics. However, adherence to the procedure outlined, coupled with careful construction techniques should produce microstrip circulators useful over the desired frequency range.

This outline consists of six individual procedures.

They can be listed as follows:

1. Determine for what use the unit is needed.
2. Determine microstrip parameters and construct the unit.
3. Determine the ferrite parameters ΔH , $4\pi M_s$, R . Obtain ferrite and check parameters.
4. Determine the proper magnetic bias field.
5. Design matching structures.
6. Check isolation, insertion loss and VSWR at all ports. Check the symmetry of the device. Make final adjustments.

Each step will be discussed fully.

Use of the Desired Circulator

Circulators can be produced in a variety of shapes and sizes. They are versatile devices which can perform a number of tasks. For best results the purpose for which the circulator is to be used should be ascertained before the design of the unit is begun. This information, along with the specifications for operating characteristics, will help determine how the device should be constructed. Here it will be assumed that these specifications warrant the decision of microstrip construction. After this decision has been made the operating characteristics and tolerances must be considered. Often these operating characteristics will be specified for the design engineer. If not, a thorough knowledge of circulator phenomena may give an indication of reasonable performance goals. The factors which limit the performance of microstrip circulators should be kept in mind.

The isolation, defined by

$$(12) \quad \text{Isol} = 10 \left| \log_{10} \frac{P_{\text{out}}}{P_{\text{in}}} \right| \text{db.},$$

is a measure of how well a particular port is isolated from the input power. Specifically it can be shown that the isolation between two ports is dependent on the VSWR at the preceding port (38). For instance if circulation is in the

1-2-3-1 direction the isolation between ports 3-1 is dependent on the VSWR at port 2. Since the circulator is meant to be a symmetric device it is necessary to match all ports. One method of providing this match will be discussed in detail later. Another factor related to the matching of the ports is the bandwidth of the device. It is not difficult to obtain a bandwidth of 20 per cent or less. Wider bandwidths will require special matching techniques. The bandwidth of a 3-port circulator is usually taken as that operating range over which the isolation is greater than 20 db.

The insertion loss of the device is the total power dissipated in the device, including microstrip and ferrite losses. This is usually related to the total power available at the input port of the device and is expressed in decibels.

For this program the following specifications were given:

$$f_0 = 4000 \text{ MHz}$$

$$\delta = 20\%$$

$$\text{Isolation} > 20 \text{ db over the bandwidth}$$

$$\text{Insertion Loss} < .6 \text{ db over the bandwidth}$$

The results verify the merits of the procedure here outlined. All of the circulators were constructed by placing a ferrite disk in a dielectric substrate. Other methods have been proposed (21, 24) but were not tested. Construction capabilities dictated the above choice.

Microstrip Parameters

Microstrip is an unbalanced transmission line. Figure 2. It was first discussed in 1952 (6). At that time an approximate analysis was made assuming that the microstrip could support TEM mode propagation. Under this assumption an electrostatic approach can be used to obtain the value of the characteristic impedance Z_0 .

For TEM mode propagation

$$(13) \quad Z_0 = \frac{\sqrt{\mu\epsilon}}{C}$$

where μ is the permeability of the substrate, ϵ is the permittivity of the substrate and C is the capacitance of the line defined by

$$(14) \quad C = \frac{Q}{(\phi_2 - \phi_1)}$$

where Q is the charge per unit length on either conductor and $(\phi_2 - \phi_1)$ is the potential difference between the conductors. Since microstrip will not support a pure TEM mode the characteristic impedance Z'_0 of microstrip is related to the above characteristic impedance Z_0 by

$$(15) \quad Z'_0 = \frac{1}{\sqrt{\epsilon_{\text{eff}}}} Z_0$$

where ϵ_{eff} is the effective relative dielectric constant.

This effective relative dielectric constant is a weighting factor which takes into account the fact that the top conductor lies in a plane dividing two different dielectrics -- air, with dielectric constant ϵ_0 , and substrate with dielectric constant ϵ_r . Wheeler (46) gives the following formula for the effective relative dielectric constant

$$(16) \quad \epsilon_{\text{eff}} = 1 + q(\epsilon_r - 1)$$

where ϵ_r is the relative dielectric constant of the substrate and q is the effective filling factor.

Presser (35), using Wheeler's work as a basis, gives a simple method of finding the effective filling factor, the effective relative dielectric constant and the shape ratio $\frac{w}{h}$ to give a desired microstrip line characteristic impedance.

Also of importance are the losses inherent in the microstrip configuration. These losses are of three types, conductor loss, dielectric loss and radiation loss. The first two have received the most attention and approximate expressions for them exist. Again it is assumed that most of the rf energy is in the region between the conductors and that TEM mode propagation is present. Then the conductor loss is (6)

$$(17) \quad \alpha_c = \frac{R_{s1} + R_{s2}}{2Z_0 w}$$

where R_{s_1} and R_{s_2} are the surface resistivity in ohm per square meter and w is the width of the top conductor.

The dielectric loss is given by

$$(18) \quad \alpha_d = \frac{\omega}{2} \sqrt{\mu\epsilon} \tan\delta$$

where ω is the driving frequency, μ is the permeability of the substrate, ϵ is the permittivity of the substrate and $\tan\delta$ is the loss tangent of the substrate. It should be noted that the dielectric loss is independent of the size and shape of the microstrip.

The third contribution to microstrip loss may, in some cases be the largest, yet it is the most difficult to analyze. Although much research is now being done on microstrip the problem of radiation has not received much attention. These radiation losses are present, particularly near sharp corners or transitions. To reduce the effect of radiation losses careful construction procedures should be used. Also the conductor surface must be kept smooth and homogeneous, and any transitions which are unavoidable should be made with extreme care. Since testing of microstrip components usually entails transitions from microstrip to coax a method of making these transitions will be described. A versatile easy-to-connect transition which could be used over a wide frequency range and with a variety of substrate materials was desired. It seemed that an in-line transition,

Figure 7, best satisfied these characteristics.

Several sizes of connectors were experimented with and it was found that the smallest was the least lossy. This was attributed to a decrease in radiation from the coax connector.

In all cases the center conductor of the connector was allowed to protrude $1/8$ ". This was flattened and pressed against the microstrip line. Spot soldering was sometimes used if there was any doubt about the press contact.

If the microstrip line width was much wider than the center conductor of the transition element the ends of the conductors were trimmed to a symmetric arrowhead configuration with an angle of approximately 30° . Figure 8. This was necessary in order to remove the conductor from the outer structure of the coaxial adaptor. The angle of 30° was selected empirically because it gave the best frequency characteristics over the bandwidth.

It was found that another critical area was the abutment of the dielectric substrate and the dielectric insulation between the coax conductors. An air gap at this point caused discontinuities which could not be tolerated at the working frequencies. After the connections were made the dielectric was placed on a piece of triangular aluminum which served as the ground plane and the holder of the single pole ceramic magnet. It was necessary to seat the dielectric firmly on the aluminum ground plane to avoid any unrepeatable variations.

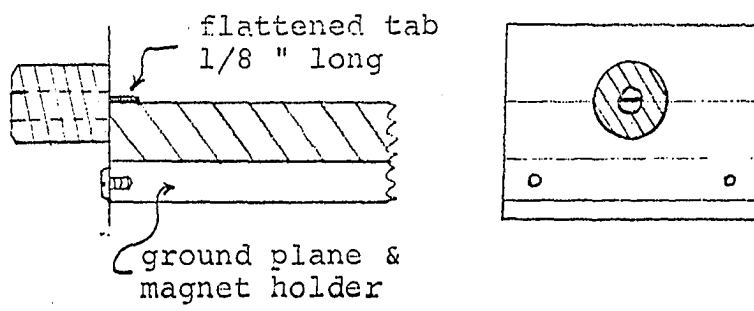


Figure 7. In-line transition from coax to microstrip

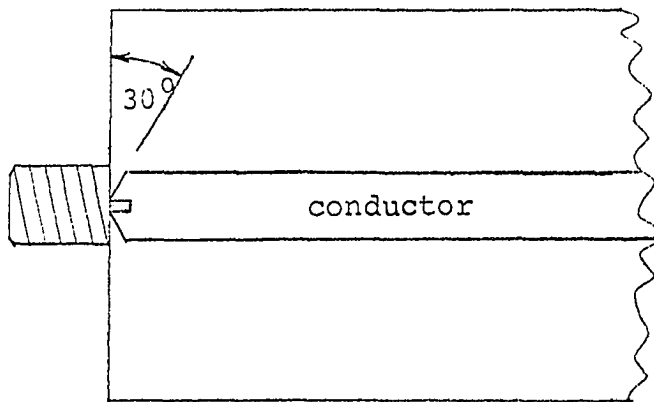


Figure 8. Microstrip conductor geometry near end of lines

Even though care was used in the fabrication of the in-line transition it was impossible to rid the structure of frequency dependent characteristics. It was, however, possible to obtain a VSWR of less than 1.16 over the frequency range of interest. Figure 9.

Hopefully many of these transition problems will be avoided in the ultimate circuit configuration of the ferrite circulator. It is envisioned as an integral part of a planar circuit with only input and output connections.

Some other comments pertaining to the choice of dielectric substrate should be made here. First the final overall size of the dielectric substrate should be selected before the ferrite is imbedded. Although a larger model might be easier to work with in the early experimental stages it is not an easy matter to scale the circulator down in size while preserving the same operating characteristics.

Another point that must be mentioned is the symmetry of the device. This is not a particularly simple objective to obtain in microstrip construction. The conductor geometry atop the ferrite must maintain three fold symmetry. A number of different conductor geometries was tested. Figure 10. These different geometries effect the circulator operating characteristics, especially the operating frequency, in a manner which has not yet been explained fully. The first conductor geometry used was the straight Y. This gave good characteristics but it was mechanically difficult to maintain

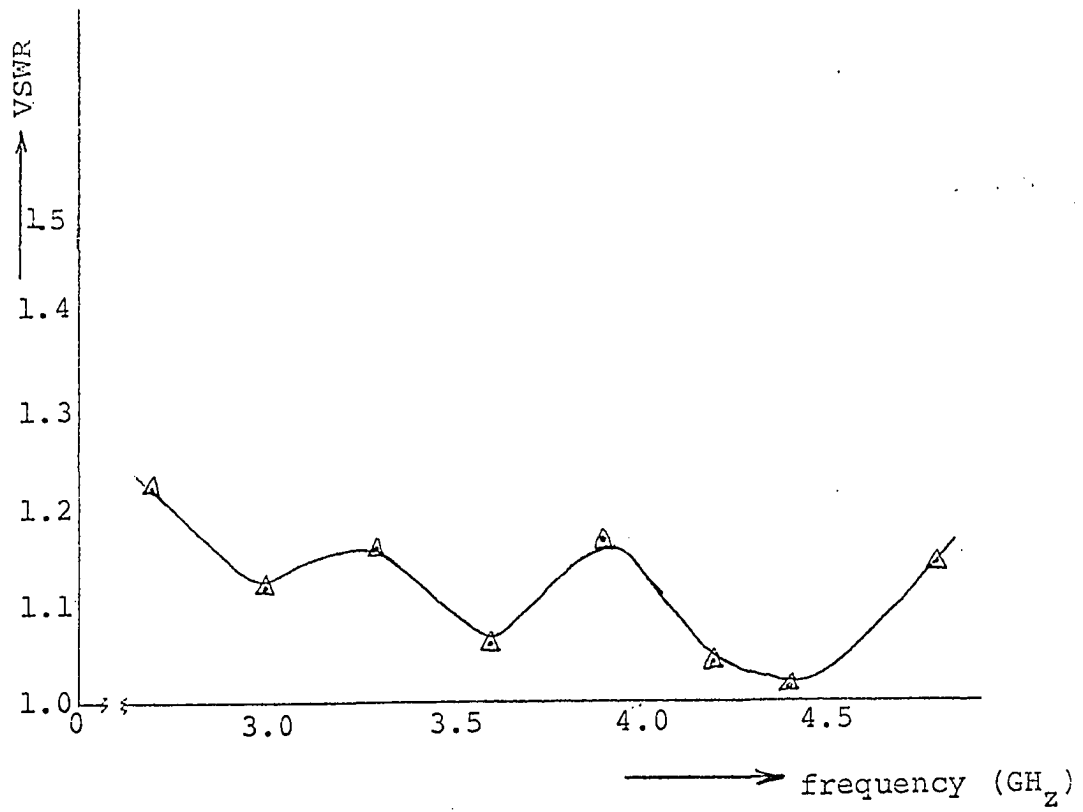


Figure 9. Input VSWR versus frequency

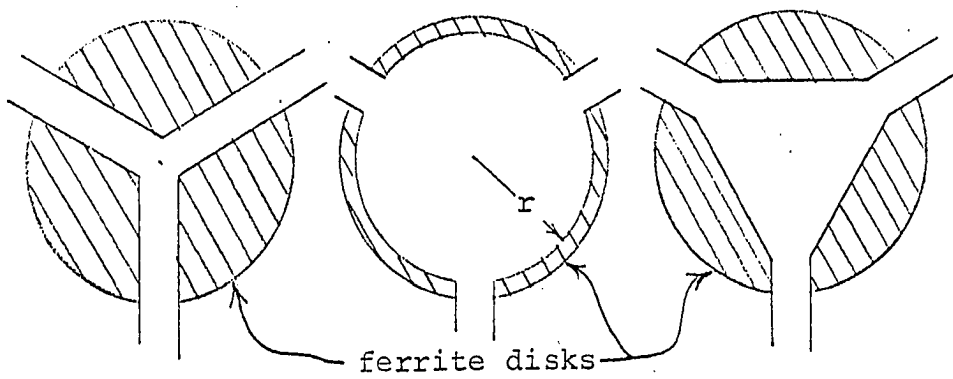


Figure 10. Different conductor geometries used on ferrite disk

symmetry. Circular disks of various radii were then used on the ferrite. It was observed that the operating frequency could be varied by changing the conductor disk diameter. This proved to be a factor in the final adjustment of the circulator.

In order to gain some insight into the effect of conductor disk radius r on the operating frequency of the circulator, a rough model of the problem was proposed and some calculations were made. (Appendix) The simplified example was that of a dielectric cylinder having field configurations such that $\underline{n} \times \underline{E} = 0$ on the top and bottom and $\underline{n} \times \underline{H} = 0$ on the sides. For TM^z modes and no z variation the fields can be written as:

$$(19) \quad H_{\phi}^0 = -k_{\rho} J_1'(k_{\rho} \rho) e^{-j\phi}$$

$$H_{\rho}^0 = -\frac{j}{\rho} J_1(k_{\rho} \rho) e^{-j\phi}$$

$$E_z^0 = \frac{k^2}{\hat{Y}} J_1(k_{\rho} \rho) e^{-j\phi}$$

where $k^2 = \omega_0^2 \mu_0 \mu'_{eff} \epsilon_0 \epsilon_{eff}$

and $\hat{Y} = j\omega_0 \epsilon_{eff} \epsilon_0$.

If we now assume that the conductor on the top of the dielectric is shrunk in size ($r < R$) and that on the exposed dielectric $\underline{n} \times \underline{H} = 0$, with all other boundary conditions remaining the same we can write new field expressions

$$(20) \quad E_{\rho}^p = \frac{-Ak_{\rho}\pi}{2Yd} J_1'(k_{\rho}\rho) \sin \frac{\pi z}{2d} e^{-j\phi}$$

$$E_{\rho}^p = \frac{jA\pi}{2Yd} \frac{1}{\rho} J_1(k_{\rho}\rho) \sin \frac{\pi z}{2d} e^{-j\phi}$$

where A is a constant which constrains the amplitudes of the perturbed fields to be of the same order of magnitude as the original fields. Note that these tangential fields have z dependence.

Considering the change in r as a perturbation of the original system an expression for the change in operation frequency can be written as

$$(21) \quad \omega - \omega_0 = \frac{\int_{\text{top}}^{\text{perturbed}} (H_{\phi}^* E_{\rho}^p - H_{\rho}^* E_{\phi}^p) \cdot d\underline{s}}{2 \int_{\text{volume of cylinder}} (\epsilon E_z^0 \cdot E_z^*) d\tau}$$

where the superscripts indicate original or perturbed fields. These integrals can be evaluated for various values of r/R and the graph of Figure 18 (Appendix) can be plotted.

Several points of interest should be noted about this graph. First as $r \rightarrow R$, $\omega \rightarrow \omega_0$ as it must. Also as r decreases the operating frequency decreases. This might not seem intuitively correct at first glance. If one considers a decrease in conductor radius as a decrease in shunt capacitance only, he might expect an increase in resonant frequency. That this is not so indicates the changes are more complex than just decreasing the capacitance of Figure 6.

Although this is a rough approximation it indicates the trend to be expected when a change in the conductor radius r is made. As seen from Figure 18 a sizable change in operating frequency can be observed. This change was verified in the laboratory when working with both strip-line and microstrip circulators. The conductor configuration is certainly a factor to be considered in any circulator construction program.

A more thorough treatment using the field expressions for a magnetized ferrite disk should increase the effectiveness of the above technique.

For the C-band unit discussed here the conductor radius was the same as the ferrite radius. The only circulator theory available is for this configuration.

A triangular plate conductor was also experimented with but proved to reduce the bandwidth over which the circulator could be matched.

Ferrite Parameters

After determination of the specifications which must be met and after the microstrip parameters have been chosen the ferrite material must be selected. The specifications of the ferrite which will be discussed are the saturation magnetization $4\pi M_s$, the line width ΔH , and the disk geometry. These quantities must be related to the desired operating

characteristics of the device. Another factor to be considered for construction purposes is the thickness of the dielectric substrate to be used. It is desirable to have this substrate and the ferrite disk with approximately the same thickness if the ferrite is to be inset in the substrate.

It was found that the thickness of the disk is not a sensitive parameter. Therefore elaborate calculations need not be made to determine an exact thickness. Any effect made by a variation in disk thickness can easily be compensated for by adjustments in the external circuitry. For instance, conductor geometry proved to be a sensitive factor in this regard.

The fact that disk thickness is not critical points out that so many assumptions have been made in the theoretical analysis of a complex phenomena that the model is not adequate in all instances.

However, in any design plan a launching point must be found. In the design used here many of the assumptions associated with the strip-line model have been adopted, some with qualifications. Also some assumptions were based upon experience gained by working with strip-line circulator models.

Experimentally it was found possible to obtain similar operating characteristics by using the same ferrite in both strip-line and microstrip models. This indicates that similar phenomena is occurring in the ferrite loaded junction region. Therefore, it was not surprising that the design

procedure of Fay and Comstock (22) gave reasonable results when adapted to microstrip. Before proceeding to the calculation of the ferrite disk diameter a discussion of two ferrite parameters of interest to the circulator designer should be given. These parameters are the saturation magnetization $4\pi M_s$ and the linewidth, ΔH .

The saturation magnetization is that value of the magnetization in which all of the magnetic dipoles are aligned with and precessing about the applied magnetic field vector at 0°K . For temperatures greater than 0°K the thermal energy will not permit complete alignment of all magnetic dipoles. Therefore, the magnetization measured at room temperature will be less than $4\pi M_s$ and will be designated $4\pi M_0$.

The fact that $4\pi M_0$ is temperature dependent proves to be a problem for the ferrite device which is to operate over wide temperature variations. Some improvement in temperature stability has recently been made in some mixed ferrites but if the temperature variations are severe external temperature control must be used.

The saturation magnetization is a determining factor in the choice of the external magnetic field strength. For low loss operation the ferrite should be near saturation. The external field across the ferrite which will keep it saturated is dependent on the saturation magnetization and on the shape of the material. The demagnetizing or shape factors can be introduced by the following example. Suppose a ferrimagnetic

sample is placed in an external field H_{dc} . This external field produces a magnetization $M_0 < M_s$ in the sample. The magnetic dipoles induced at the surface of the sample create a magnetic field which opposes the applied field. This demagnetizing field strength is dependent on the value of $4\pi M_0$ for the material. The microscopic field internal to the sample will be given by

$$(22) \quad H_i = H_{dc} - N(4\pi M_0)$$

where N , the demagnetizing factor is dependent on the shape of the sample. Graphs are available for evaluating the demagnetizing factors for common sample geometries (32).

For ferrite circulator operation it is desirable to have the internal field equal zero. This then gives the first indication of the external field which will be required.

$$(23) \quad H_{dc} = N(4\pi M_0), \quad M_0 \approx M_s$$

(In c.g.s. units there is no distinction between the units of field strength, oersteds, and the units of flux density, gauss.)

If the applied field is not strong enough to produce a magnetization which is near the value needed for saturation the ferrite losses will increase due to the non-alignment of the magnetic dipoles. This is the so called low field loss

region which must be avoided for efficient circulator operation. Figure 11. In the class of microstrip circulators where a single pole piece is to be used to provide the external field it becomes important to choose a ferrite material with a low saturation magnetization. Values of $4\pi M_0$ from 300 to 600 gauss are common.

Another parameter of importance is the line width ΔH (oersteds) of the ferrite material. The line width is a measure of the time rate of energy exchange between the precessing dipoles and the lattice vibrations of the ferrite material. Values of line width may range from about one oersted for Yig to several hundred oersteds for some ferrites. A narrow line width material indicates a slow rate of energy exchange and a small damping of the magnetization vector. This narrow line width material is desired for ferrite circulators. However, extremely narrow line width material is very expensive and fortunately not required for efficient circulator action. Values from 50 to 200 oersteds are common.

The saturation magnetization and the line width are two parameters which are usually specified when a ferrite material is delivered. If there is a need to measure these parameters several measurement techniques using thin disks can be found (5, 8, 36, 37). With some changes this same method can be used with ferrite samples of different shapes.

Having chosen a ferrite material with suitable values of

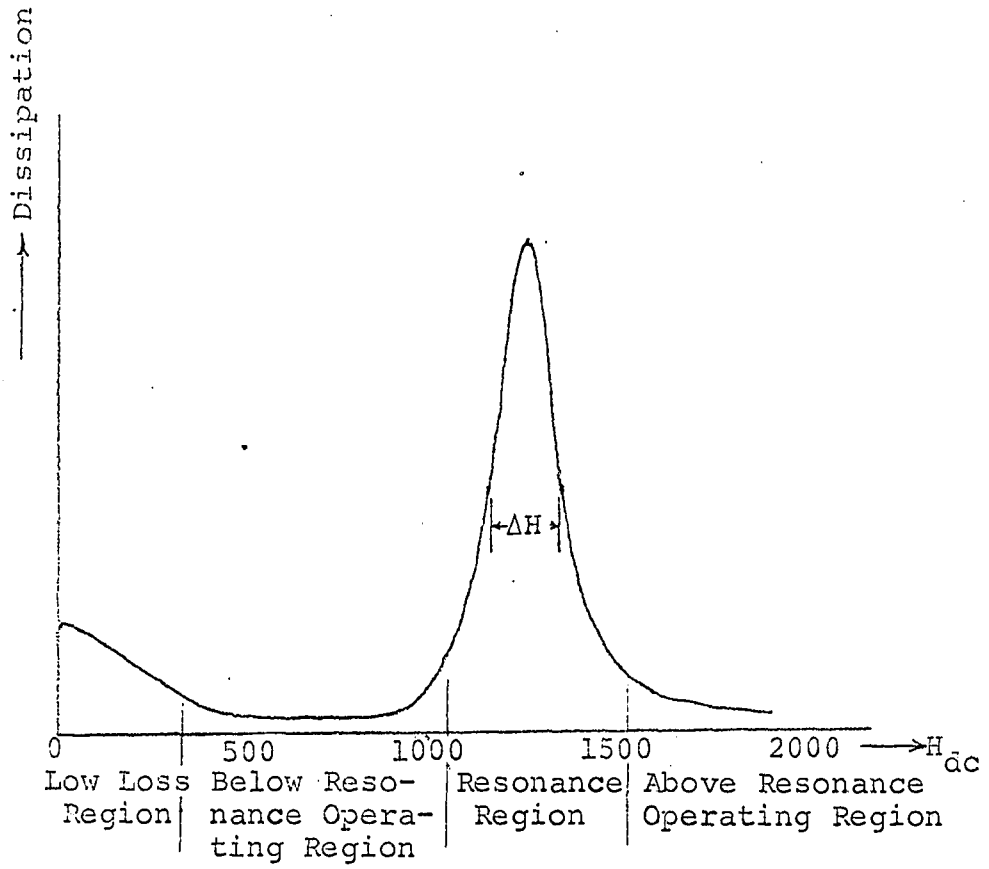


Figure 11. Resonance curve showing different regions of operation for a fixed rf frequency

$4\pi M_s$ and ΔH the final parameters of interest are the dimensions. The proper choice of disk diameter will insure that at the desired center frequency, f_0 , the admittance of the disk will be real, or nearly so. The disk thickness is a minor controlling factor on the bandwidth of the device. In microstrip it is not a critical dimension and for this program it was chosen to be the same dimension as the substrate thickness.

To calculate the disk diameter the procedure proposed by Fay and Comstock (22) was used. The notation is:

$\delta = \frac{1}{2}$ bandwidth (in per cent)

VSWR = voltage standing wave ratio at input port at band edge

Y_R = admittance of disk resonator at band edge (see Figure 6)

θ = phase angle of Y_R

Q_L = loaded Q of disk resonator

δ' = fractional frequency splitting required for an admittance phase angle of 30° at the resonator.

Quarter-wave transformer matching techniques are to be used. It will be desirable if the susceptance of the transformer will cancel the susceptance of the resonator at the band edges. If this is true then for small bandwidth (<30%) the

$$(24) \quad \text{VSWR} \approx \sec^2 \theta = \frac{Y_R^2}{G_R^2} .$$

Having found θ the loaded Q of the resonator can be solved for using

$$(25) \quad Q_L = \frac{\tan \theta}{2\delta} .$$

This allows the fractional frequency splitting, δ' to be evaluated from

$$(26) \quad \delta' = \frac{\tan 30^\circ}{2Q_L} .$$

From this it is possible to find the ratio $\frac{K}{\mu}$ from

$$(27) \quad \frac{K}{\mu} \approx 2.46\delta'$$

where the value of the constant is evaluated by considering Equation 9 assuming that for small splitting the relation of δ' to $\frac{K}{\mu}$ is linear. It can then be shown (22) that if the internal field is zero in the ferrite

$$(28) \quad \mu_{\text{eff}} \approx 1 - \left(\frac{K}{\mu}\right)^2 .$$

Since this design is for microstrip, where a composite dielectric is present, the effective relative permeability must be altered similar to finding an effective relative permittivity as discussed earlier. To do this, Wheeler's work (47) was used as a basis. Given the μ_{eff} as just

calculated it is possible to find an expression for μ_{eff} in terms of the same effective filling fraction q . This is

$$(29) \quad \mu'_{\text{eff}} = \frac{\mu_{\text{eff}}}{q + \mu_{\text{eff}}(1 - q)} .$$

For the usual range of values of μ_{eff} this will prove not to be a large correction.

Using this value and the value of effective permittivity the value of wave number can be evaluated from

$$(30) \quad k = \omega \sqrt{\epsilon_{\text{eff}} \mu'_{\text{eff}} \mu_0 \epsilon_0}$$

and this value can be used in

$$(31) \quad (kR)_{1,1} = 1.84$$

to evaluate the ferrite disk diameter for the lowest order modes.

Using the specifications given for this program and a ferrite with $4\pi M_0 \approx 450$ gauss the disk diameter was calculated to be 0.54 inches.

Determination of the Proper Bias Field

It is assumed that the microstrip and ferrite parameters have been chosen, the materials have been obtained, and a symmetrical unit constructed. Consideration must now be

given to the third basic problem -- that of providing the proper magnetic bias field. In microstrip this is usually done by placing a single magnetic pole under the ferrite. These pole pieces can be obtained commercially in several shapes and sizes. The obtainable magnetic field strength varies widely with magnet material and size.

It has already been shown that the field strength should be approximately

$$(32) \quad H_{dc} \approx N(4\pi M_0).$$

Some adjustment of the magnetic field strength applied to the ferrite can be gained by using a set of metallic shims to vary the distance between magnet and ferrite. All magnetic field measurements were made by placing the probe of a hall effect gaussmeter on top of the microstrip conductor, directly above the center of the ferrite disk.

All theory assumes that the ferrite is completely saturated by a uniform external magnetic field. Due to the fact that no magnetic material can be placed above the conducting lines without affecting the electric field the required magnet is often larger than would be necessary in other circulator models. In fact, experimental evidence indicates the magnet pole piece must have at least 20 per cent larger diameter than the ferrite to decrease edge effects. The insertion loss increased if smaller magnets were used. This was

attributed to nonparallel alignment of magnetic dipoles near the ferrite disk edge.

For a given operating frequency f_0 , the variation of the bias field changes the admittance of the ferrite and the insertion loss of the circulator. The typical ferromagnetic resonance curve (Figure 11) with frequency as a fixed parameter can be used as a visual aid when selecting the proper bias field. Obviously, for below resonance operation, the desirable region is between the low field loss region and the ferromagnetic resonance loss region. This curve, available with most commercially procured ferrites, proved to be deceptive in that it indicated a wide range of values of magnetic bias.

Experimentally it was found that the range of bias field values which gave minimum insertion loss was much more narrow. Figure 12. Choosing a value of magnetic bias which minimizes the insertion loss may not be wise, however, until the effect of the bias field on the admittance versus frequency plots is investigated.

In order to make admittance measurements an effective reference position was needed. This was provided by substituting a metal slug for the ferrite. This provided a short circuit reference for the slotted-line impedance measuring technique.

A typical admittance plot is shown in Figure 13 for several different bias field values. It is seen that as the

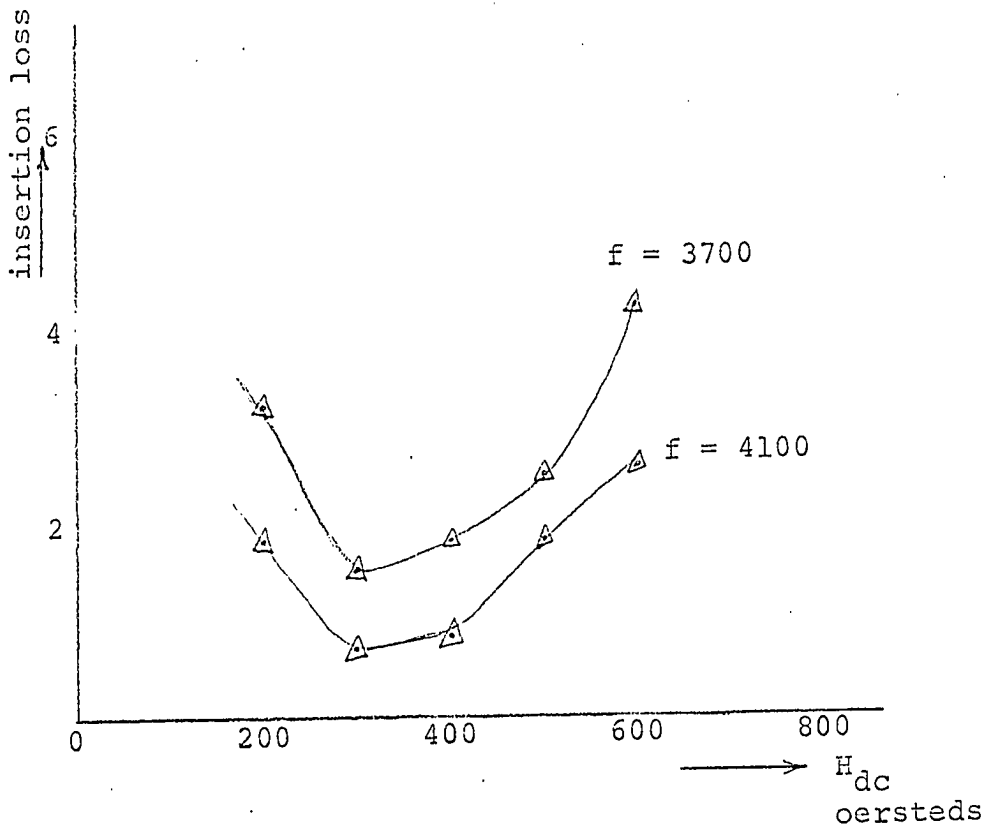


Figure 12. Insertion loss versus H_{dc}

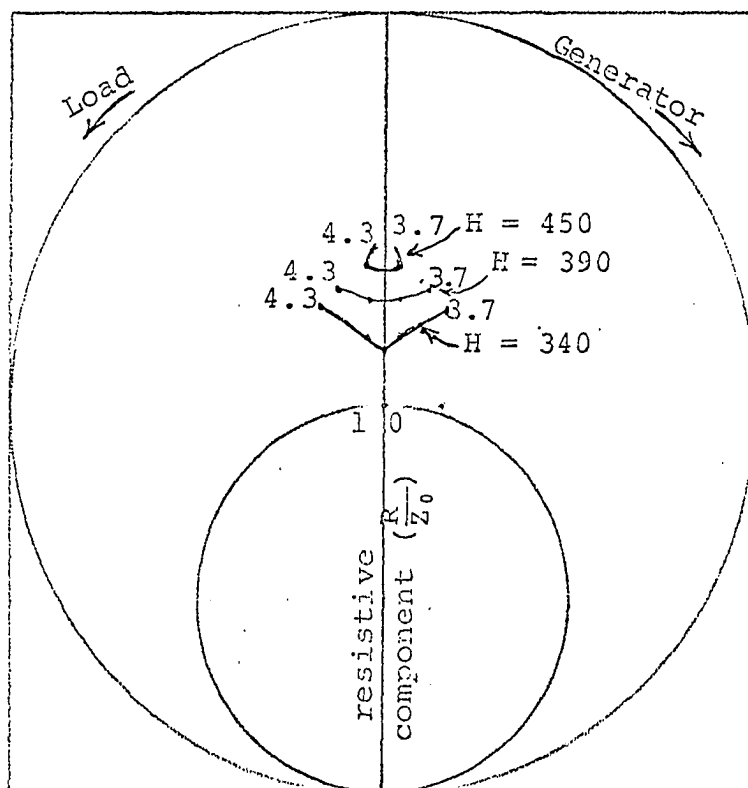


Figure 13. Port admittance plot

applied field is increased the port admittance increases. However the bandwidth over which the admittance is essentially constant also increases. Note also that the band edge frequencies are brought closer to the real axis of the Smith chart. This allows matching with relatively simple quarter wave transformers.

Considering the data obtained in the laboratory a decision on the optimum bias field can be made. The particular specifications to be met will effect the decision as to emphasizing minimum insertion loss or a more matchable admittance plot.

Matching the Ports

It is known that the isolation obtained from the circulator is dependent on the port match provided. Since a symmetrical device is desired, all ports should be matched to the same degree. As mentioned above quarter-wave transformers were used to lower the admittance of the resonator to that of the microstrip line and to cancel the susceptance of the resonator at the band edges.

Having chosen a bias field and having an admittance versus frequency plot to refer to, the transformer can be designed using established techniques (34). In microstrip the proper line width to obtain a desired transformer impedance is most easily found using graphical techniques (35). The length of the transformer is to be a quarter wavelength

at the center frequency. One problem encountered was where to measure the length of the transformer. When taking data for the admittance plots it was necessary to terminate the microstrip in a short circuit. This was done by inserting a metal slug in place of the ferrite. The impedance plot was then based on this reference. Since the line subtends an angle of 2ψ on the ferrite it was decided to make the transformer-ferrite connection such that an angle of 2ψ was subtended here also. Figure 14. The width of this connection does affect the match since it changes the effective length of the transformer. This proved to be an extremely sensitive area of the structure. Small variations in conductor geometry near the ferrite altered the port admittance and the matching structures were then not as efficient.

Experimentation with various conductor disk radii produced the following empirical pattern. As the conductor disk radius was reduced the center operating frequency decreased. In other words the conductor covering the ferrite changes the admittance presented to the port in such a way that the admittance versus frequency plot of Figure 13 is shifted circularly to the left. An explanation of this effect is given elsewhere in this report.

Indeed there are other variables available which effect the port match. For instance the conductor geometry need not be a circular disk. As long as three-fold symmetry is maintained other geometries can be used which will effect the

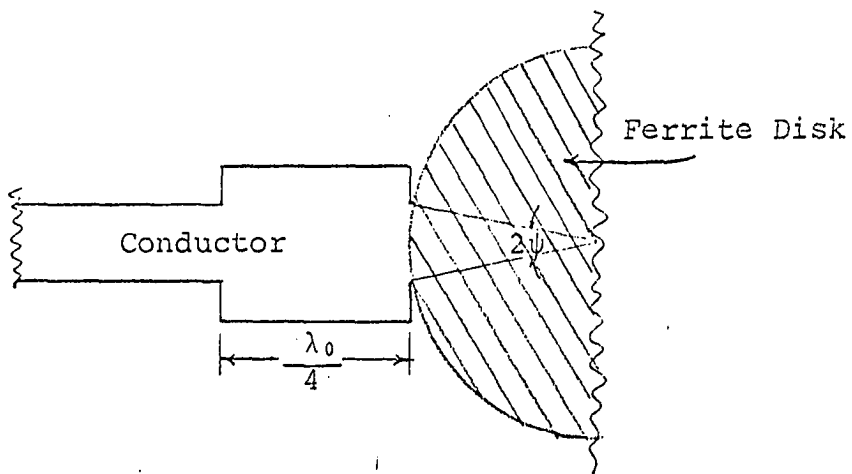


Figure 14. Transformer-ferrite connection

admittance and the match. Figure 10. Unfortunately, these effects are unpredictable theoretically due to the approximations used in describing the complex phenomena of circulation.

It is wise to refer to the admittance versus frequency plot obtained previously and carefully construct all matching networks. It will usually be possible to improve the match obtained on the first try. This is not an exact art.

Measurement of Circulator Characteristics

Developing significant testing procedures is, of course, necessary in any program of this type. The typical workbench is shown in Figure 15 in block diagram form. With this set-up the following information could be obtained: a) the isolation (reverse power flow) between any two ports, b) the total insertion loss of the system, c) VSWR at any port, and d) the admittance versus frequency plot for the junction. The first three measurements are made on the completed circulator model. The admittance versus frequency plot was discussed previously. The isolation measurement is easily made using a network analyzer and sweeping the frequency through the operating range. The insertion loss measurement was made using a substitution method. This method is valid and good results are obtained if there are no large mismatches in the system. In the final run when the ports are matched this condition is satisfied. The VSWR's at each port should be measured and compared. Any asymmetry will be discovered by

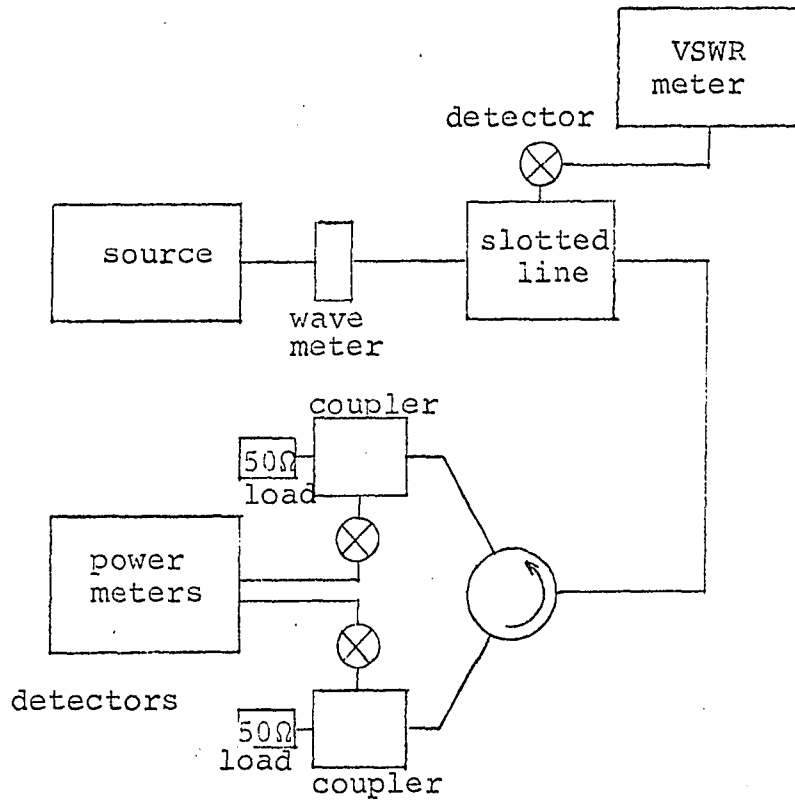
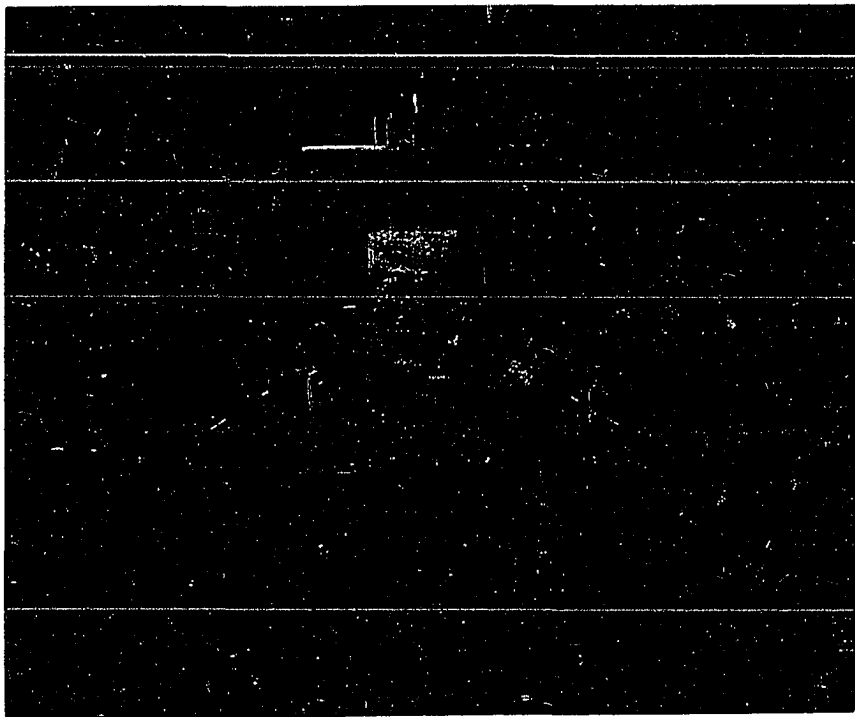


Figure 15. Necessary laboratory equipment for complete testing program

making this comparison. This problem can be solved by changing slightly the width or length of the microstrip line to that port.

One of the C-band microstrip ferrite circulators constructed and tested during this study is shown in Figure 16. The substrate was triangular and measured $1\frac{1}{4}$ " on a side. The complete unit including magnet weighed 13 ounces. The characteristics of this particular unit are given in graphical form in Figure 17.

Figure 16. Picture of complete microstrip circulator



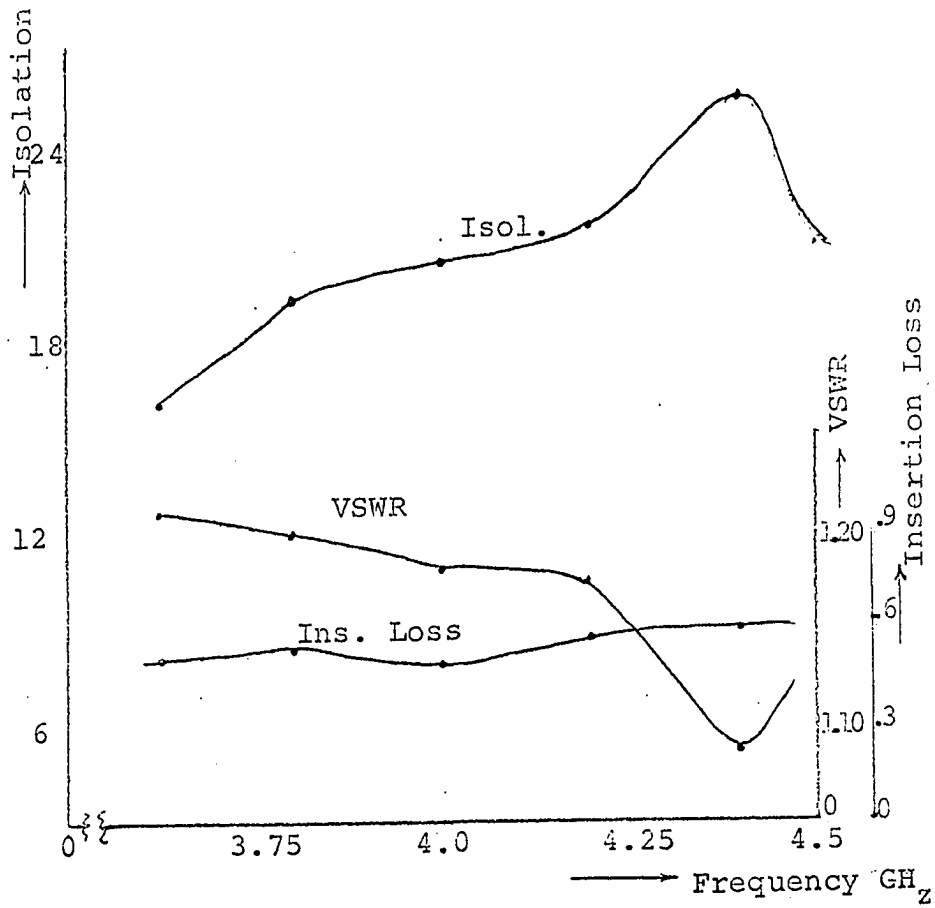


Figure 17. Characteristics of C-band microstrip circulator

CONCLUSIONS

Microstrip circulator design is not a simple clear-cut process. A considerable amount of engineering man-hours lie between the design and the realization of a device which will meet those objectives. A reduction of these engineering man-hours is the prime objective of this study.

The engineer is concerned with the optimum method of meeting certain design specifications. A thorough understanding of all aspects of the problem is essential to the engineer, if he is to choose wisely among the alternative approaches to the final solution. The "best" solution usually involves compromises. For this reason a review of both the theoretical and the experimental work which preceded this undertaking is included in this report. Although most of this work was done in waveguide or strip-line it is necessary to understand the similarities and the differences between these circulators and the ones constructed using a microstrip transmission system.

Experimental work with both strip-line and microstrip circulators indicated that similar phenomena were taking place in both junction areas. Therefore, a design plan for microstrip, based on an available analysis of the strip-line circulator, was formulated. This design plan was then carried through to the ultimate construction of a C-band microstrip circulator using a ferrite disk imbedded in a dielectric

substrate. Although this is the most common construction method, other proposed methods, such as using a ferrite substrate, are thought to be adaptable to many of the techniques outlined here.

The engineering aspects of the circulator problem were not neglected. Some of these problems, which are discussed in detail, are transitions from microstrip transmission line to coaxial transmission line, port matching, proper magnetic bias field selection and application, and device symmetry. Also, since it was found experimentally that the operating characteristics of the circulator could be varied by changing the conductor configuration on top of the ferrite, an elementary theoretical explanation was offered.

In summary, this is the most comprehensive paper yet written on the engineering considerations involved in the designing of microstrip circulators. However the field is far from closed! The area of microwave ferrite devices is presently the subject of an extensive research effort. Much work needs to be done on such topics as ferrite deposition, the effect of conductor geometry on the operating characteristics, a more efficient manner of applying the necessary magnetic bias, and the temperature stability of the ferrite device. It is hoped that, in time, the microstrip circulator will be a dependable component of a complete planar system.

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ACKNOWLEDGMENTS

The author would like to thank his major professor, Dr. A. V. Pohm, and his entire graduate committee for the guidance of his graduate study program. Special thanks goes to Dr. Robert E. Post for his many helpful suggestions concerning this research effort. Much of this experimental work was done while the author was working for Collins Radio Company. The author would like to thank his colleague, Mr. Ed Hudock, for the many helpful discussions they held during this period. Finally, the author would like to extend his thanks to the faculty and graduate students of Iowa State University with whom he has associated during the last three years.

APPENDIX

The effect of conductor geometry on the circulator operating characteristics is not fully understood. It has been found experimentally that a sizeable change in operating frequency can be caused by changing the radius of the conducting disk on top of the ferrite. This also causes less noticeable changes in port admittance and optimum magnetic bias field value.

To gain some insight into the problem of operating frequency alteration a much simplified model was proposed (see page 45) and some calculations were made.

This appendix will deal with the evaluation of Equation 21 which is rewritten here for convenience and numbered 1A.

$$(1A) \quad \omega - \omega_0 = \frac{-j \int_{\text{perturbed}} \int (H_{\phi}^0 E_{\rho}^D - H_{\rho}^0 E_{\phi}^D) \cdot ds}{2 \int_{\text{volume}} \int \int \epsilon E_z^0 \cdot E_z^0 d\tau} = \frac{\text{numerator}}{\text{denominator}}$$

Solving the numerator we have

$$(2A) \quad \frac{-j\pi A}{2\bar{y}d} \left\{ \frac{1}{k_{\rho}^2} \int_{k_{\rho}r}^{k_{\rho}R} \int_0^{2\pi} x (J_1'(x))^2 d\phi dx + \int_{k_{\rho}r}^{k_{\rho}R} \int_0^{2\pi} \frac{1}{x} (J_1(x))^2 d\phi dx \right\}$$

$$= \frac{-\pi^2 A}{\bar{y}d} \left\{ \int_{k_{\rho}r}^{k_{\rho}R} x (J_1'(x))^2 dx + \int_{k_{\rho}r}^{k_{\rho}R} \frac{1}{x} (J_1(x))^2 dx \right\}.$$

Using the identities

$$(3A) \quad J_1'(x) = J_0(x) - \frac{1}{x} J_1(x)$$

and

$$(4A) \quad J_1(x) = \frac{x}{2} (J_0(x) + J_2(x))$$

these can be arranged as

$$(2A') \quad -235 \left\{ \frac{1}{2} \int_{k_\rho r}^{k_\rho R} x (J_0(x))^2 dx + \int_{k_\rho r}^{k_\rho R} x (J_2(x))^2 dx \right\}.$$

The integrals can be evaluated using

$$(5A) \quad \int x \{J_n(\alpha x)\}^2 dx = \frac{x^2}{2} \{J_n'(\alpha x)\}^2 + \frac{x^2}{2} \left(1 - \frac{n^2}{\alpha^2 x^2}\right) \{J_n(\alpha x)\}^2.$$

After some manipulation the numerator becomes

$$(2A'') \quad -235 \left\{ k_\rho (R-r) J_0 \{k_\rho (R-r)\} J_1 \{k_\rho (R-R)\} + \{J_1(k_\rho (R-r))\}^2 \right. \\ \left. \left(\frac{k_\rho^2 (R^2 - r^2)}{2} - 1 \right) \right\}.$$

The denominator can be evaluated as follows:

$$(6A) \quad 2\epsilon \int_0^R \int_0^d \int_0^{2\pi} |E_z|^2 d\phi dz \rho d\rho = \frac{4\pi\epsilon\omega^2\mu^2 d}{k_\rho^2} \int_0^{k_\rho R} x (J_1(x))^2 dx.$$

Using 7A to evaluate the integral gives

$$(6A') \quad (4 \times 10^{-8}) \left\{ \frac{k_{\rho}^2 R^2}{2} \{ (J_0(k_{\rho} R))^2 - \frac{2}{k_{\rho} R} (J_0(k_{\rho} R) J_1(k_{\rho} R)) + (J_1(k_{\rho} R))^2 \} \right\}.$$

Evaluating this for a disk radius of 6.85×10^{-3} meters gives for the denominator

$$(6A'') \quad \text{Denominator} = 1.63 \times 10^{-8}.$$

Now evaluating the numerator for several values of $\frac{r}{R}$ the graph of Figure 18 can be drawn.

It is thought that, although the model proposed is elementary, the trend which emerged gives an indication of observable laboratory behavior.

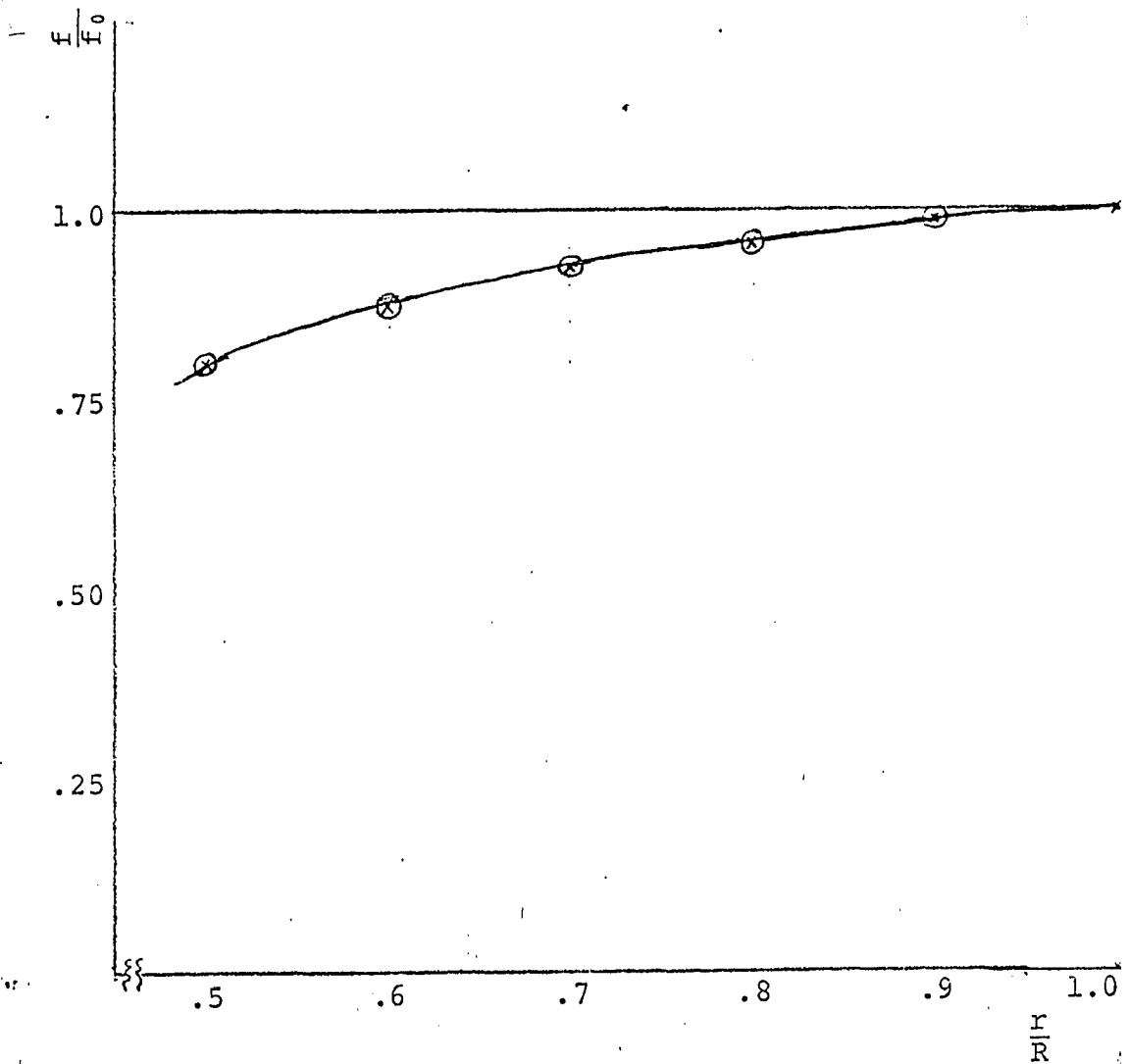


Figure 18. Variation of operating frequency with conductor disk radius